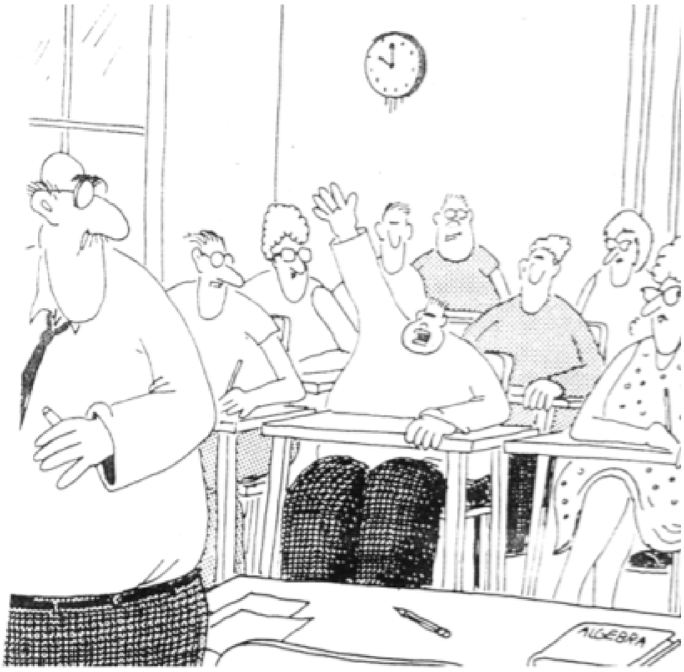
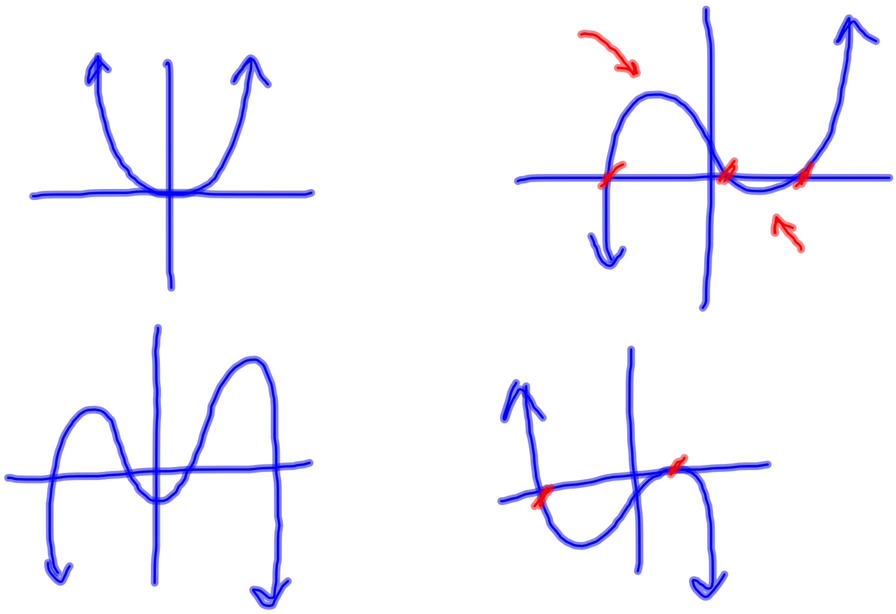


"Mr. Hargreaves, may I be excused?
My brain is full"

Polynomials

The "BIG" Picture



- end behavior
- roots, zeros, intercepts
- multiplicity
- turning pts.

Graphs of Polynomial Functions

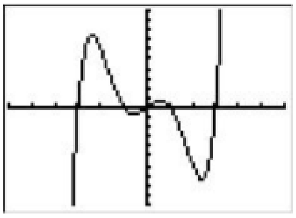
- Continuous:

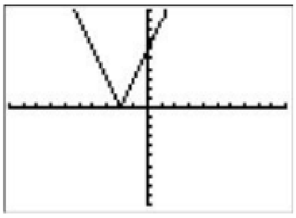
no breaks, gaps, or holes

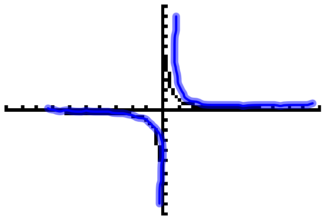
$\frac{1}{x}$

- Smooth turns and bends – no sharp turns.

Ex:







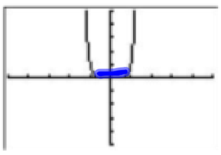
X X

$2x^5 + 3x^2 - 11$

Simplest Polynomials, $f(x)=x^n$

Power function

- When n is even:

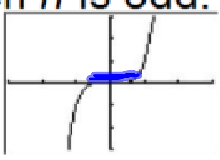


D:
R:
Increasing on:
Symmetry:

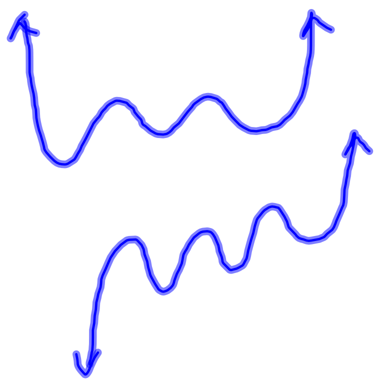
$\uparrow n$

$\rightarrow$ more flexed

- When n is odd:

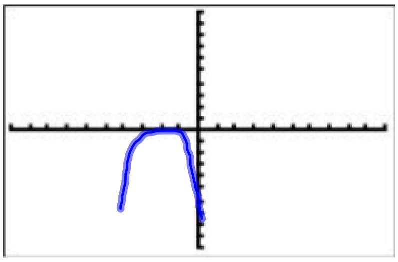


D:
R:
Increasing on:
Symmetry:

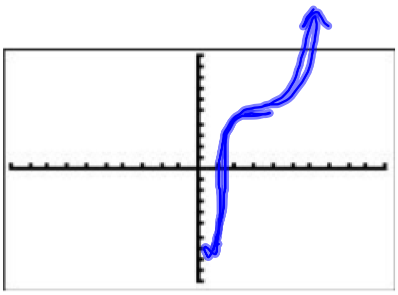


"flex"

a) $f(x) = -(x + 2)^4$



b) $f(x) = (x - 3)^5 + 4$



The Leading Coefficient Test

Describe the relationship between the degree and the leading coefficient of the function and the right-left-hand behavior of the graph of the function:

- M $Y = x^3 - 2x^2 - x + 1$
- T $Y = 2x^5 + 2x^2 - 5x + 1$

L↓ R↑

- K $Y = -2x^5 - x^2 + 5x + 3$
- L $Y = -x^3 + 5x - 2$

L↑ R↓

- A $Y = 2x^2 + 3x - 4$
- C $Y = x^4 - 3x^2 + 2x - 1$

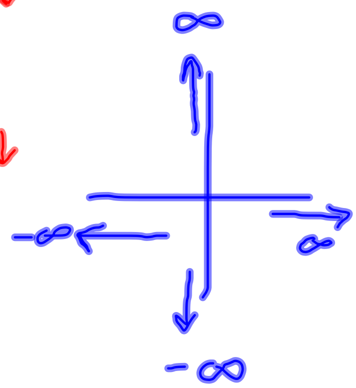
L↑ R↑

- I $Y = -x^2 + 3x + 2$
- B $Y = -x^6 - x^2 - 5x + 4$

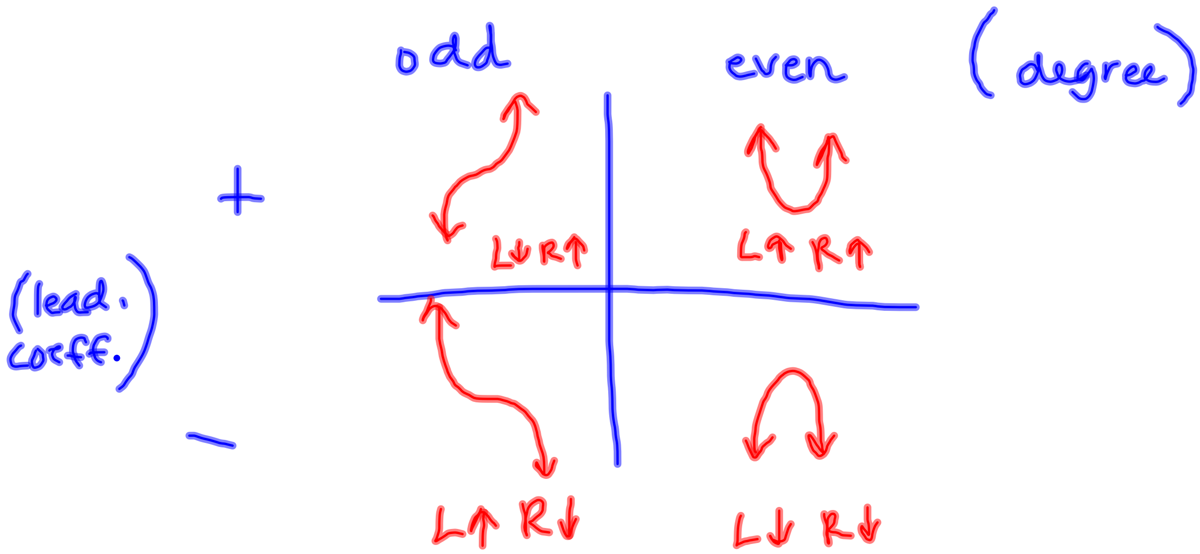
L↓ R↓

$f(x) = a^n x^n + \ldots$	pos $a^n > 0$ Leading	neg $a^n < 0$ coeffecient
n is even	$x \to -\infty$ $f(x) \to \infty$ L↑ $x \to \infty$ $f(x) \to \infty$ R↑	$x \to -\infty$ $f(x) \to -\infty$ L↓ $x \to \infty$ $f(x) \to -\infty$ R↓
n is odd	$x \to -\infty$ $f(x) \to -\infty$ L↓ $x \to \infty$ $f(x) \to \infty$ R↑	$x \to -\infty$ $f(x) \to \infty$ L↑ $x \to \infty$ $f(x) \to -\infty$ R↓

degree



end behavior



Ex 2) Describe the right-hand and left-hand behavior of the graph of each function:

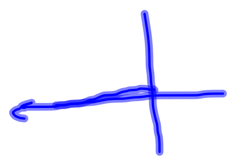
a) $f(x) = -x^4 + 7x^3 - 14x - 9$

L $x \to -\infty$ $f(x) \to -\infty$

L↓ R↓

R $x \to \infty$ $f(x) \to -\infty$

b) $g(x) = 5x^5 + 2x^3 - 14x + 6$



L↓ R↑

Zeros of Polynomial Functions

It can be shown that for a polynomial function of degree n, the following statements are true:

1. The function f has at most n real zeros.

   n zeros

2. The graph of f has at most n-1 relative extrema.

   turning pts.

Zeros of Polynomial Functions

If f is a polynomial function and a is a real number, the following statements are equivalent:

zero → sol → x-int.

1) $x = a$ is a *zero* of the function f.

2) $x = a$ is a *solution* of the polynomial equation $f(x) = 0$.

3) $(x - a)$ is a factor of the polynomial $f(x)$.

4) $(a, 0)$ is an x-intercept of the graph of f.

Find all real zeros of:

& graph

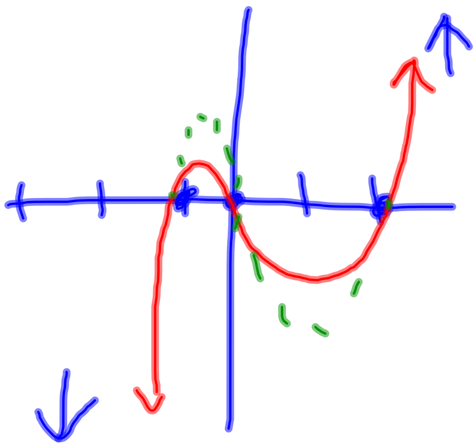
$f(x)= x^3 - x^2 - 2x$

$x^3-x^2-2x=0$

$x(x^2-x-2)=0$

$x(x-2)(x+1)^2=0$

$x=0, 2, -1$



Repeated Zeros

For a polynomial function, a factor of $(x - a)^k$, $k > 1$, yields a repeated zero $x = a$ of multiplicity k.

1) If k is **odd**, the graph *crosses* the x-axis at $x = a$.

2) If k is **even**, the graph touches the x-axis (but does not cross the x-axis) at $x = a$. (bounces)

Sketching the graph of a polynomial

1. Find end behavior

2. Find the zeros

2.5 Determine muetiplicity of zeros

3. Plot a few extra points

4) Sketch the graph of $f(x) = -x^3 - 2x^2$

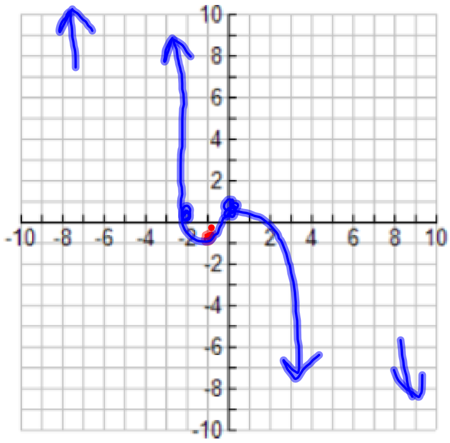
$-x^3 - 2x^2 = 0$

$-x^2(x+2) = 0$

even mult.   odd mult.

$x = 0, -2$

bounces   crosses



L ↑

R ↓

x	f(x)
-1	-1

Sketch the graph of f(x)= $x^4 - 5x^2 + 4$

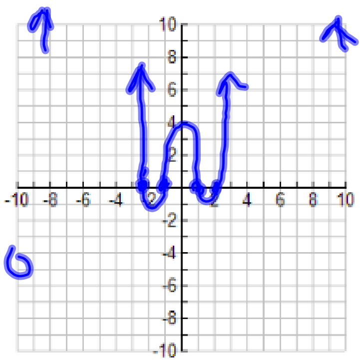
$x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$(x-2)(x+2)(x-1)(x+1) = 0$

* all odd multiplicity

$x = 2, -2, 1, -1$



L ↑

R ↑

Find a polynomial function with the following zeros.
0, -1, 3, with a degree of 3

$$x(x+1)(x-3)$$

$$x(x^2-2x-3)$$

$$\boxed{f(x) = x^3 - 2x^2 - 3x}$$

Intermediate Value Theorem:

see pg. 282

Use the Imtermediate Value Theorem to approximate the real zero of:

$$f(x) = x^3 - x^2 +1$$

HW:

Pg 284 #1-8, 10,17,18,34-37, 59,68,73,89